1995-96 KIRIS OPEN-RESPONSE ITEM SCORING WORKSHEET

Grade 11 — Mathematics Question 4

The academic expectations addressed by this item include:

1.5 - 1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.12 Students, understand mathematical structure concepts including the properties and logic of various mathematical systems.

The core content assessed by this item includes:

Algebra Concept
- Students should understand linear, quadratic, and exponential equations and functions.

Algebra Skill
- Students should be able to solve and graph a variety of equations and inequalities.
- Students should be able to construct tables of numeric values of equations and inequalities.

Algebra Relationship
- Students should understand the following relationships: how formulas, tables, graphs, words, and equations of functions relate to each other.

4. Best Price for Greatest Income

Steve has started a small business selling a regional newsletter. He has 500 subscribers who pay $8.00 for a subscription. Steve has tried raising the prices. For each $0.25 he raises the subscription price, he loses 10 customers. Help Steve determine what price he must charge in order to receive the greatest gross income. Show all of your work.

SCORING GUIDE

Score	Description
4	Identifies $10.25 as the price generating the greatest income; shows or explains a complete strategy for determining the correct price.
3	Computational error leads to an incorrect price; appropriate strategy shown OR Correct price, $10.25, with a vague or incomplete justification.
2	Correct price, $10.25, with no or incorrect attempt at justification of price OR Some appropriate reasoning for determining earnings charging varied prices and decreasing number of sales.
1	Minimal understanding; shows little or no understanding beyond finding the income for the current price and number of sales. One correct pair multiplied.
0	Incorrect answer with no valid procedure.
Blank	Blank/no response.

(500 - 10X)(8 + 0.25X)

500•8.00	= 4,000
490•8.25	= 4,042.50
480•8.50	= 4,080
470•8.75	= 4,112.50
460•9.00	= 4,140
450•9.25	= 4,162.50
440•9.50	= 4,180
430•9.75	= 4,192.50
420•10.00	= 4,200
410•10.25	**= 4,202.50**
400•10.50	= 4,200

KIRIS ASSESSMENT ANNOTATED RESPONSE
GRADE 11 MATHEMATICS

Sample 4-Point Response of Student Work

Student correctly identifies $10.25 as the subscription price that would generate the greatest income.

If Steve subtracts 10 subscribers for every $0.25 he raises his subscription, he would still keep making an increase in profit until he has only 410 customers. By then he would be charging $10.25 for a subscription. He would be making $4,202.50. That is $202.50 more than he started. If he charges more and 10 more quit subscribing he would be losing money.

500 customers
x $8 for a subscription
4000

490	480	460	420	~~380~~	410
x8.25	x8.50	x$9	x$10	~~x$11~~	x$10.25
4042.5	4080	4140	4200	~~4180~~	4202.5

400
x10.50 can't go any higher than $10.25
4200

Student relates words and calculations to the underlying quadratic equation $y = (500 - 10x)(8. + .25x)$.

Student's listing of numeric values of the equation skips over the domain, then zeroes in on the maximum of the quadratic equation.

Response includes a complete explanation, using trial calculations that show gross income increasing at a reasonable sampling of subscription prices until price reaches the point of diminishing return.

KIRIS ASSESSMENT ANNOTATED RESPONSE
GRADE 11 MATHEMATICS

Sample 3-Point Response of Student Work

Explanation is minimal and contains a minor error: the price for greatest profit is $10.25, which is $2.25 (not $2.50) over the original price.

If Steve drops 90 customers and raises the price by $2.50 he will gain $202.50

#of customers	Times	Pay
490	8.25	4042.50
480	8.50	4080.00
470	8.75	4112.50
460	9.00	4140.00
450	9.25	4162.50
440	9.50	4180.00
430	9.75	4192.50
420	10.00	4200.00
410	10.25	4202.50
400	10.50	4200.00

Table shows correct combinations of subscribers, prices, and income.

KIRIS ASSESSMENT ANNOTATED RESPONSE
GRADE 11 MATHEMATICS

Sample 2-Point Response of Student Work

Student identifies $9.00 as a subscription price that would generate **greater**, but not **maximum**, profit.

He should charge $9.00 for a 12 week subscription. He would make more money charging 460 customers $9, than he would changing 500 customers $8.

$$\begin{array}{rr} {\scriptstyle 5} & \\ 460 & 500 \\ \underline{\quad 9} & \underline{\quad 8} \\ \$4{,}140.00 & \$4{,}000.00 \end{array}$$

Reasoning and approach are appropriate and correct, but not carried far enough to yield correct answer to question.

Sample 1-Point Response of Student Work

Student works only with numbers given in problem; does not demonstrate understanding of purpose of question.

For Steve to make the greatest gross income between $8.00 or $8.25 he needs to charge the 8.25.

500 - 8.00 - 4,000
490 - 8.25 - 4,082.50

Student's work does not show evidence of finding numeric values of the underlying quadratic function $f(x) = (500 - 10x)(8 + .25x)$, showing the increase and then decrease of gross income, and then identifying the maximum.

INSTRUCTIONAL STRATEGIES

Best Price for Greatest Income

Examine algebraic representations, compare linear and quadratic equations, graphs, and relationships.

Examine maximum and minimum, compare area versus perimeter of figures.

Organize work with a table, chart, graph.

Give students experiences working problems in which they have to decipher information to solve the problem.

Use problems like this to help students develop methods to write an equation from given information, create a chart, develop their own method to solve a problem.

Use KIRIS-like open response questions in classroom instruction and assessment. Model strategies for explaining work to fellow mathematicians. Model and have students develop and use scoring guides with open response items. Encourage students to explore highlighting and underlining strategies as organizers, stressing that only evidence found in Student Response Book is scored on KIRIS open responses.

Infuse lessons with the use of a variety of instructional approaches and strategies:

- use mathematical tools, manipulatives, hands on activities, cooperative group work, higher order thinking skills, video tapes, multiple intelligences approaches, mappings, graphic organizers, etc.

Explore appropriate use of calculators, both as tools and instruments for checking work.

REFERENCES

TRANSFORMATIONS Kentucky's Curriculum Framework
Academic Expectations 1.5-1.9 and 2.7 through 2.13
KDE's Core Content for Assessment
Mathematics, examine curriculum alignment from P through 12
KDE's web site at http://www.kde.state.ky.us
explore curriculum pages, examine units of study, etc.
Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics, Assessment Standards for School Mathematics, and Addenda Series from NCTM.
Telephone: 703-620-9840, web site at http://www.nctm.org